

Meson Decays in an Extended Nambu–Jona-Lasinio model with Heavy Quark Flavors

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Abstract

In a previous work, we proposed an extended Nambu–Jona-Lasinio (NJL) model including heavy quark flavors. In this work, we will calculate strong and radiative decays of vector mesons in this extended NJL model, including light ρ , ω , K^* , ϕ and heavy D^* , D_s^* , B^* , B_s^* .

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I. INTRODUCTION

The Nambu–Jona-Lasinio (NJL) model [1, 2], in its original form as a pre-QCD theory, was constructed of nucleons that interact via an effective two-body contact interaction. Later the model was reinterpreted as a theory of quark degrees of freedom [3, 4]. The most important feature of NJL model is the chiral symmetry of Lagrangian plus a chiral symmetry breaking ground state. The model was generalized to $SU(3)_f$ case of light quark flavors in refs. [5–9].

On the other side, for heavy quark flavors the chiral symmetry no longer holds. However, new important symmetries such as the spin symmetry were discovered in heavy $(Q\bar{q})$ -mesons [10], which is a consequence of the order $1/m_Q$ of spin-spin interaction in the effective quark potential [11]. In ref. [12], The NJL model was generalized to include heavy flavors. Both the chiral symmetry in light meson sector and the spin symmetry in heavy meson sector were reproduced with the vector-current interaction. The bosonization technique was used there to obtain an effective Lagrangian of meson degrees of freedom.

However as already shown in ref. [5], vector-current interaction itself is not enough to reproduce the experimental masses of light vector mesons such as ρ , K^* etc. Other chiral symmetrical interactions such as the axial-vector-current one, are needed to get satisfactory results for light meson sector. But these additional interactions do not obey the spin symmetry in heavy meson sector since they will generate the incorrect spin-spin interaction that is not $1/m_Q$ suppressed. In the above work [12], the authors just introduced two coupling constants G_1 and G_2 for the light meson sector and another different coupling G_3 for the heavy meson sector.

In our previous work [13], we proposed a solution to extend the NJL model to comprise the heavy quark flavors. The NJL interactions were expanded with respect to $1/m_f$ of constituent quark mass m_f just like the expansion in the heavy quark effective theory (HQET). Naturally the vector-current interaction is dominant while other interactions such as the typical axial-vector-current one should be $1/m_f$ suppressed. We had performed numerical calculations for both the light and heavy meson sectors. The mass spectra fit the experimental data quite well. The decay constants of heavy mesons were smaller than experimental values roughly by a factor of 2.

The strong and radiative decays provide us important information about hadron struc-

ture. Experimentally, the decay widths of light vector mesons have been well measured [14–19] and so far some of decay widths or ratios of the charmed and bottomed heavy vector mesons were reported [20–22].

Generally speaking, it is a rigid test for any model to fit the experimental values of decay width or ratio. The most popular model for strong decay is the 3P_0 model [23, 24]. This model has been applied to a great number of decay processes [25–28]. The radiative decays, mainly M1 transition which takes place when one of the constituent quark changes its spin and radiates one photon, has been studied in potential quark models [29, 30] or from flavor symmetry [31]. For decays of heavy mesons, abundant works have been done in the frameworks of chiral quark model [30, 32], potential model [33, 34], bag model [35], chiral perturbation model [36], and QCD sum rules [37, 38]. The decays were also studied in NJL model [39, 40] and from lattice QCD [41–43].

In this work, we will calculate strong and radiative decays of vector mesons in the extended NJL model with heavy flavors, including light mesons ρ , ω , K^* , ϕ and heavy ones D^* , D_s^* , B^* , B_s^* .

II. MODEL AND FORMALISM

In ref. [13], the Nambu-Jona-Lasinio model was generalized to deal with heavy quarks as well as light ones. The Lagrangian reads

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - \hat{m}_0)\psi + \mathcal{L}_4, \quad (1)$$

where

$$\mathcal{L}_4 = G_V(\bar{\psi}\lambda_c^a\gamma_\mu\psi)^2 + \frac{h}{m_q m_{q'}}[(\bar{\psi}\lambda_c^a\gamma_\mu\psi)^2 + (\bar{\psi}\lambda_c^a\gamma_\mu\gamma_5\psi)^2], \quad (2)$$

describes the four-point quark-quark interaction compatible with QCD chiral symmetry. G_V , of dimension (mass) $^{-2}$, and the dimensionless h were parameters fixed in the spectral calculation. The second term on the right side in Eq. (2) appears as higher order correction expanded with respect to the constituent quark mass m_q similar to the HQET expansion. We can rewrite Eq. (2) in a Fierz invariant form. For the light sector, one has

$$\begin{aligned} \mathcal{L}_4^q = & \frac{4}{9}G_V[(\bar{q}\lambda_f^i q)^2 + (\bar{q}i\gamma_5\lambda_f^i q)^2] \\ & - \frac{2}{9}(G_V + \frac{h}{m_q m_{q'}})[(\bar{q}\lambda_f^i\gamma_\mu q)^2 + (\bar{q}\lambda_f^i\gamma_\mu\gamma_5 q)^2]. \end{aligned} \quad (3)$$

where λ_f^i 's are the $U_f(3)$ generators, with $\lambda_f^0 = \sqrt{\frac{2}{3}}I$ (where I is the 3×3 unit matrix) and the rest are Gell-Mann matrices in flavour space. For the heavy sector, one has

$$\begin{aligned} \mathcal{L}_4^Q = & \frac{8}{9}G_V[(\bar{Q}q)^2 + (\bar{Q}i\gamma_5\lambda_f^i q)(\bar{q}i\gamma_5\lambda_f^i Q)] \\ & - \frac{4}{9}(G_V + \frac{h}{m_q m_Q})[(\bar{Q}\gamma_\mu q)(\bar{q}\gamma^\mu Q) + (\bar{q}\gamma_\mu\gamma_5 q)(\bar{q}\gamma^\mu\gamma_5 Q)], \end{aligned} \quad (4)$$

where still we have

$$Tr \lambda_i \lambda_j = 2\delta_{ij}. \quad (5)$$

One can see that actually we only consider the higher order $1/m_q m_Q$ suppressed interaction in vector and axial-vector channels and so the important chiral symmetry breaking vacuum (the ground state) is unchanged.

Using Bethe-Salpeter equation (BSE), we obtained the meson masses via the corresponding T-matrix where the mesons appear as the poles of the T-matrix. The meson-quark coupling constants were also obtained by further expanding the T-matrix around the meson poles.

In this work, we will use the effective meson Lagrangian to calculate strong and radiative decays of vector mesons. The effective meson-quark coupling constants will be directly taken from our previous work. In the cases of pseudo-scalar meson and vector meson, the corresponding effective quark couplings read

$$\mathbb{L}_{\pi q} = -g_{\pi q} \bar{\psi} i\gamma_5 \boldsymbol{\tau} \psi \cdot \boldsymbol{\pi} - \frac{f_{\pi q}}{m_\pi} \bar{\psi} \gamma_\mu \gamma_5 \boldsymbol{\tau} \psi \cdot \partial^\mu \boldsymbol{\pi}, \quad (6)$$

$$\mathbb{L}_{\rho q} = -g_{\rho q} \bar{\psi} \gamma_\mu \boldsymbol{\tau} \psi \cdot \boldsymbol{\rho}^\mu. \quad (7)$$

For the decay of a vector meson (V) into two pseudo-scalars (P), one has

$$\Gamma(V \rightarrow PP) = \frac{1}{2m_V} \int d\phi^{(2)} |\mathcal{M}(V \rightarrow PP)|^2, \quad (8)$$

where $\int d\phi^{(2)} = \int \frac{d^3 k_1}{(2\pi)^3 2E_{k_1}} \frac{d^3 k_2}{(2\pi)^3 2E_{k_2}} (2\pi)^4 \delta^4(q - k_1 - k_2)$ is the standard two-body phase-space-measure. In the rest frame of the decaying meson, the decay amplitude of the vector meson can be write as

$$\mathcal{M}(V \rightarrow PP) = \epsilon^\mu T_\mu = -\boldsymbol{\epsilon} \cdot \boldsymbol{T}, \quad (9)$$

where ϵ^μ is the polarized vector of V meson. Then we have

$$\Gamma(V \rightarrow PP) = \frac{k_c}{24\pi m_V^2} |\boldsymbol{T}|^2. \quad (10)$$

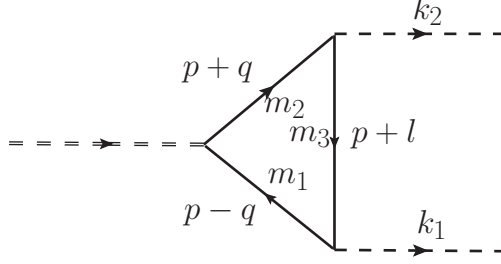


FIG. 1: The Feynman diagram corresponds to the strong decay process.

The strong decay process of a vector meson is shown in Feynmann diagram Fig. 1, where $q = \frac{k_1+k_2}{2} = (\frac{m_V}{2}, \mathbf{0})$, $l = \frac{k_1-k_2}{2} = (\frac{k_1^0-k_2^0}{2}, \mathbf{k}_c)$, and m_1, m_2, m_3 denote the constituent masses of the constituting quarks. Using the Feynman rules, one can write down the expression for the decay amplitude directly. One finds

$$iT^\mu = -Tr \int \frac{d^4p}{(2\pi)^4} ig_V \gamma^\mu \lambda^V \frac{i}{\not{p} - \not{q} - m_1} i(g_1 + \frac{\tilde{g}_1}{m_1 + m_3} \not{k}_1) i\gamma_5 \lambda^{P_1} \\ \times \frac{i}{\not{p} + \not{l} - m_3} i(g_2 + \frac{\tilde{g}_2}{m_2 + m_3} \not{k}_2) i\gamma_5 \lambda^{P_2} \frac{i}{\not{p} + \not{q} - m_2}. \quad (11)$$

For the reaction of a vector meson decays into a pseudo-scalar and a photon (γ), $V \rightarrow P\gamma$, the decay width can be expressed as

$$\Gamma(V \rightarrow P\gamma) = \frac{1}{2m_V} \int d\phi^{(2)} |\mathcal{M}|^2, \quad (12)$$

where the decay amplitude should take the form

$$i\mathcal{M}(V \rightarrow P\gamma) = e\epsilon^\mu(V)\epsilon^{*\nu}(\gamma)T_{\mu\nu}. \quad (13)$$

The Feynman diagrams of radiative decay are shown in Fig.2. We can write down the radiative decay amplitude

$$T^{\mu\nu} = Tr \int \frac{d^4p}{(2\pi)^4} ig_V \gamma^\mu \lambda^V \frac{i}{\not{p} - \not{q} - m_1} i\hat{Q}\gamma^\nu \frac{i}{\not{p} + \not{l} - m_1} \\ \times i(g_P + \tilde{g}_P \frac{\not{k}_2}{m_1 + m_2}) i\gamma_5 \lambda^P \frac{i}{\not{p} + \not{q} - m_2} \\ + Tr \int \frac{d^4p}{(2\pi)^4} ig_V \gamma^\mu \lambda^V \frac{i}{\not{p} - \not{q} - m_1} i(g_P + \tilde{g}_P \frac{\not{k}_2}{m_1 + m_2}) i\gamma_5 \lambda^P \\ \times \frac{i}{\not{p} - \not{l} - m_2} i\hat{Q}\gamma^\nu \frac{i}{\not{p} + \not{q} - m_2}.$$

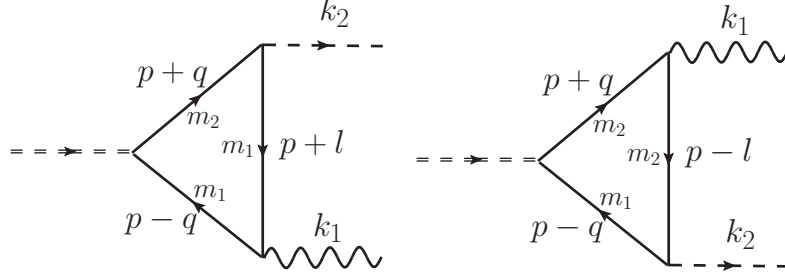


FIG. 2: The Feynman diagram corresponding to the radiative decay processes.

In the rest frame of decaying meson, we only need the space components of the tensor T^{ij} and it can be written as

$$T^{ij} = \epsilon^{ijl} T_{VP\gamma}^l. \quad (14)$$

Then we have

$$\Gamma(V \rightarrow P\gamma) = \frac{\alpha k_c}{3m_V^2} |T_{VP\gamma}|^2, \quad (15)$$

where $\alpha \simeq 1/137$ is the electromagnetic fine structure constant.

To calculate the loop integrals, we apply the three-momentum cut-off regularization scheme to the integrals. First, we define some useful quantities

$$\begin{aligned} E_p(m) &= \sqrt{\mathbf{p}^2 + m^2}, \\ E_k(m) &= \sqrt{(\mathbf{p} + \mathbf{k}_c)^2 + m^2}, \\ \omega_{1,2} &= +q^0 \pm E_p(m_1), \\ \omega_{3,4} &= -q^0 \pm E_p(m_2), \\ \omega_{5,6} &= -l^0 \pm E_k(m_3). \end{aligned}$$

The ω_i 's emerge as poles when the integral with respect to p^0 is performed. After we integrate out p^0 , the amplitudes can always be represented as spatial integrals

$$\begin{aligned} T &= \int^\Lambda \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_i^{2,4,6} \frac{N|_{p_0=\omega_i}}{\prod_{j \neq i} (\omega_i - \omega_j)} \\ &= \frac{1}{4\pi^2} \int_0^\Lambda p^2 dp \int_{-1}^1 dt \sum_i^{2,4,6} \frac{N|_{p_0=\omega_i}}{\prod_{j \neq i} (\omega_i - \omega_j)}, \end{aligned}$$

where N represents the numerator of integrand. The 2-dimensional integral will be performed numerically by Monte Carlo integration method using the **vegas** routine from **gsl** library.

TABLE I: Meson-quark coupling constants.

g_π	g_K	g_D	g_{D_s}	g_B	g_{B_s}	
4.25	4.32	4.71	5.03	5.92	6.69	
\tilde{g}_π	\tilde{g}_K	\tilde{g}_D	\tilde{g}_{D_s}	\tilde{g}_B	\tilde{g}_{B_s}	
1.56	1.61	2.04	2.09	2.84	3.11	
$g_{\rho/\omega}$	g_ϕ	g_{K^*}	g_{D^*}	$g_{D_s^*}$	g_{B^*}	$g_{B_s^*}$
1.29	1.38	1.31	1.64	1.83	2.51	2.89

III. NUMERICAL RESULTS

In the previous work [13], we had calculated the pseudo-scalar and vector mesons, light and heavy, consistently in an extended NJL model with interaction given by eq. (2). The input parameters were the current masses of light quarks and the constituent masses of heavy quarks, the two coupling constants and the 3-dimensional cutoff. Numerically, the parameters were set to

$$\begin{aligned}
 m_{u/d}^0 &= 2.79 \text{ MeV}, & m_s^0 &= 72.0 \text{ MeV}, \\
 m_c &= 1.62 \text{ GeV}, & m_b &= 4.94 \text{ GeV}, \\
 \Lambda &= 0.8 \text{ GeV}, & G_V &= 2.41, & h &= 0.65.
 \end{aligned}
 \tag{16}$$

Using above parameters we obtained the constituent masses of light quarks

$$m_u = m_d = 392 \text{ MeV}, \quad m_s = 542 \text{ MeV}.
 \tag{17}$$

The obtained meson-quark coupling constants, which we need to calculate the strong and radiative decays, are given in Table I. We will use the experimental meson masses given by Particle Data Group [44].

In Table II, we show the results for the strong and radiative decays of light vector mesons. As we can see, our results are in qualitative agreement with the empirical values. Nevertheless, quantitatively our results are systematically smaller than the empirical values by a factor of 2 or 3. The discrepancy always occurs in the NJL calculation as the model lacks the quark confinement mechanism. In the potential model [45], generally the masses of light vector mesons ρ or K^* lay above the constituent quark mass thresholds and still they are bound states due to the linear confinement potential. In our calculation, the

TABLE II: Strong and radiative decay widths for light vector mesons.

Decay modes		This work	Bernard [39]	Empirical [44]
$\rho \rightarrow \pi\pi$	MeV	68.5	52.0	149.1 ± 0.8
$\rho^\pm \rightarrow \pi^\pm\gamma$	keV	21.9	60.1	68 ± 7
$\rho^0 \rightarrow \pi^0\gamma$	keV	43.9	—	89 ± 12
$\omega \rightarrow \pi\gamma$	keV	866	762	764 ± 51
$\phi \rightarrow K^+K^-$	MeV	1.28	—	2.08
$\phi \rightarrow K_L^0 K_S^0$	MeV	0.86	—	1.46
$K^{*\pm} \rightarrow (K\pi)^\pm$	MeV	20.9	57.3	50.7 ± 0.9
$K^{*\pm} \rightarrow K^\pm\gamma$	keV	13.5	92.0	50 ± 0.5
$K^{*0} \rightarrow K^0\gamma$	keV	31.3	—	117 ± 10

constituent masses of light quarks are intentionally tune larger so that the mesons are still bound states under the constituent quark mass thresholds, even without the confinement. In another NJL calculation [39], the smaller constituent quark masses were used and the ρ and K^* vector meson was found as the resonant poles. Then they suggested to account for the discrepancy by introducing a renormalization factor of roughly 2 into the light vector meson field after have taken the higher order meson loops into consideration. In comparison, the numerical results from ref. [39] are also listed in Table II. As we know, the amplitudes of triangle Feynman Diagrams heavily depend on the quarks masses when the meson masses are close to the mass threshold. Our numerical study shows that to fit the experimental decay width of ρ demands that $2m_u$ should be very close to m_ρ and then the numerical result turns to be unstable. We guess that the confinement mechanism is important here for the light vector mesons as it is critical to their formation.

Table III shows the strong and radiative decay widths of heavy vector mesons. Table IV exhibits the branching ratios for charmed vector mesons. It can be seen that our results agree with the experimental values. As the empirical data are not complete, here we also list some of other model calculation and lattice calculation in the table for comparison. In Table III, our decay width of D^{*+} is a little lager than the empirical one. Numerically this can be corrected by changing m_c slightly, about 5 MeV larger. In Table IV, our resulted

TABLE III: Strong and radiative decay widths for heavy vector mesons (all in unit keV).

Decay Modes	This work	Kamal [46]	Goity [30]	Empirical [20, 21, 47]
$D^{*\pm} \rightarrow D^{\pm}\pi^0$	39.7	25.9	28.8	
$D^{*\pm} \rightarrow D^0\pi^{\pm}$	84.4	58.8	64.6	
$D^{*\pm} \rightarrow D^{\pm}\gamma$	0.7	1.7	1.4	
$D^{*\pm} \rightarrow \text{all}$	124.4	86.4	94.9	96 ± 22
$D^{*0} \rightarrow D^0\pi^0$	46.5	42.4	41.6	
$D^{*0} \rightarrow D^0\gamma$	19.4	21.8	32.0	
$D^{*0} \rightarrow \text{all}$	65.9	64.2	73.6	$< 2.1 \text{ MeV}$
$D_s^* \rightarrow D_s\gamma$	0.09	0.21	0.32	$< 1.9 \text{ MeV}$
$B^{*\pm} \rightarrow B^{\pm}\gamma$	0.25	—	0.74	
$B^{*0} \rightarrow B^0\gamma$	0.22	—	0.23	
$B_s^* \rightarrow B_s\gamma$	0.10	—	0.14	

TABLE IV: Branching ratios for charmed vector mesons.

Decay Modes	This work	Kamal [46]	Goity [30]	Empirical [44]
$D^{*\pm} \rightarrow D^{\pm}\pi^0$	31.8	30.0	30.3	30.7 ± 0.5
$D^{*\pm} \rightarrow D^0\pi^{\pm}$	67.7	68.0	68.1	67.7 ± 0.5
$D^{*\pm} \rightarrow D^{\pm}\gamma$	0.5	2.0	1.5	1.6 ± 0.5
$D^{*0} \rightarrow D^0\pi^0$	70.6	66.0	56.5	61 ± 2.9
$D^{*0} \rightarrow D^0\gamma$	29.4	34.0	43.5	38.1 ± 2.9

branching ratios also are in agreement with the experimental data. Here the numerical results are less sensitive to constituent quark masses than that of the light meson sector. We may expect that the calculation of strong and radiative decays for heavy mesons are more reliable as it is well known that for heavy mesons the confinement is less important than the one gluon exchange coulomb potential.

IV. SUMMARY

We have used the extended NJL model with heavy flavors [13] to calculate strong and radiative decays of vector mesons. It should be noted that no extra assumption and free parameter was introduced into our present calculation. A reasonable agreement to the experimental data is obtained. The results of light vector mesons may indicate that a more complex quark structure should be considered for vector meson due to the confinement which is lacked in NJL model.

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